http://poloclub.gatech.edu/cse6242

CSE6242 / CX4242: Data & Visual Analytics

Graphs / Networks

Basics, how to build & store graphs, laws, etc.
Centrality, and algorithms you should know

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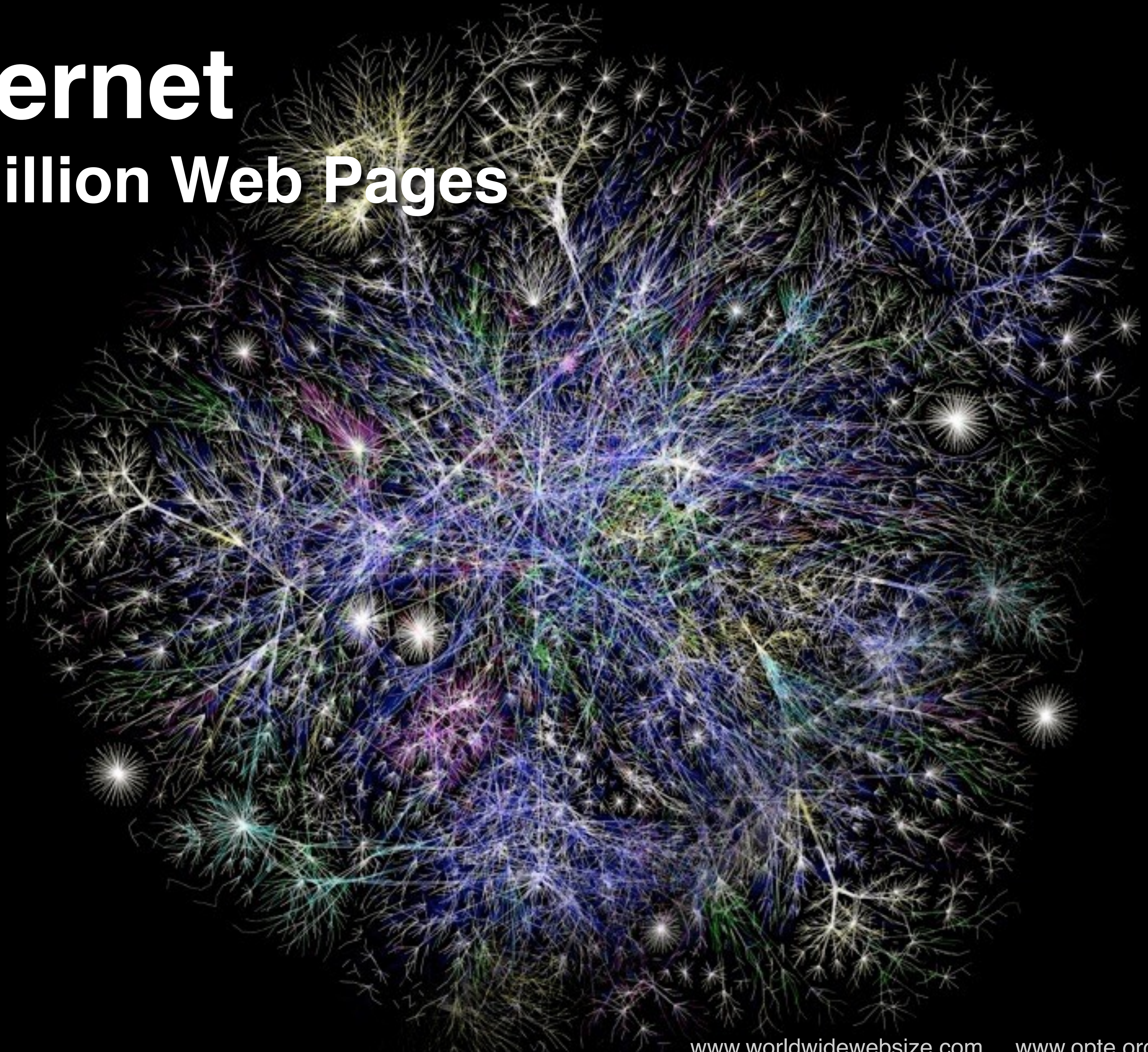
Partly based on materials by
Professors Guy Lebanon, Jeffrey Heer, John Stasko, Christos Faloutsos, Parishit Ram (GT PhD alum; IBM), Alex Gray

Internet

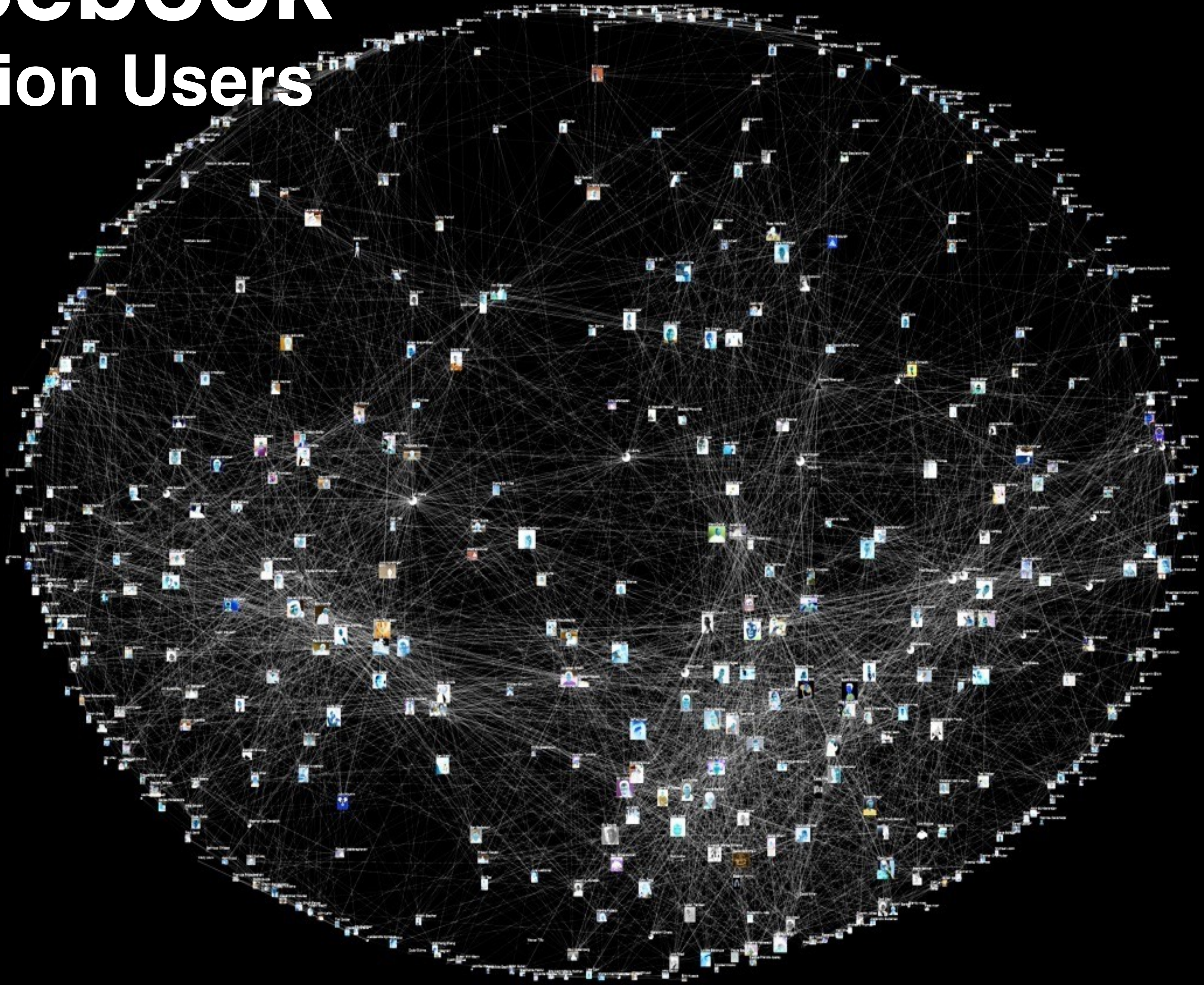
4 Billion Web Pages



www.worldwidewebsize.com www.opte.org

Facebook

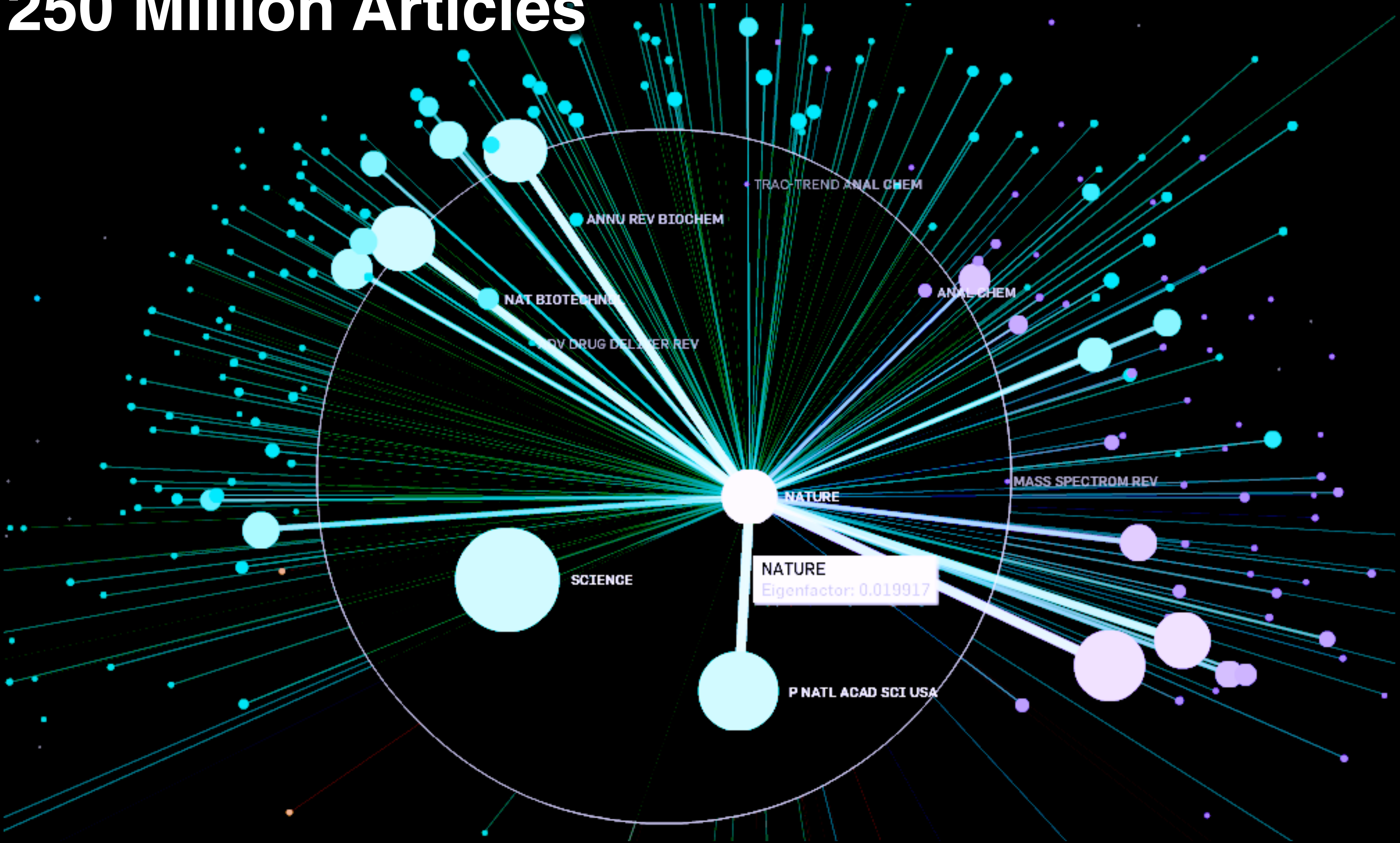
2 Billion Users



Modified from Marc_Smith, flickr

Citation Network

250 Million Articles



www.scirus.com/press/html/feb_2006.html#2 Modified from well-formed.eigenfactor.org

Many More

twitter

Who-follows-whom (288 million users)

amazon

Who-buys-what (120 million users)

at&t cellphone network

Who-calls-whom (100 million users)

Protein-protein interactions

200 million possible interactions in human genome

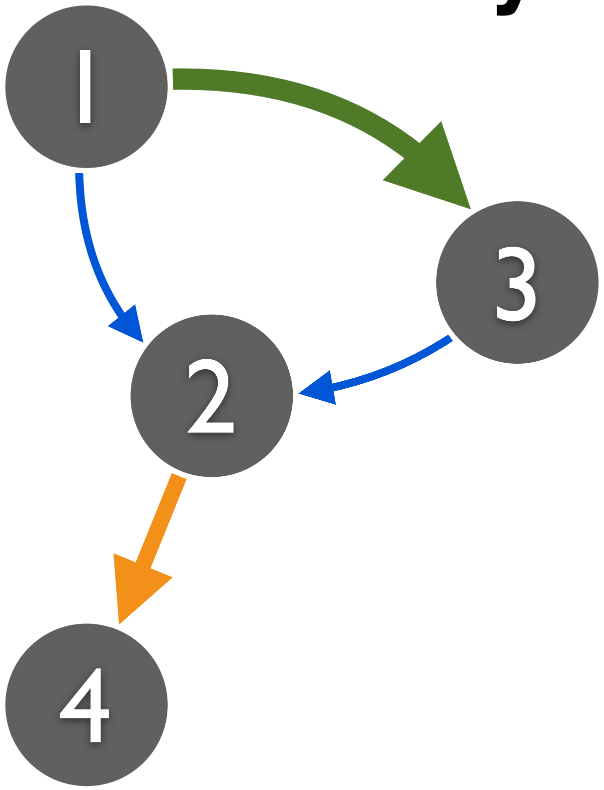
Sources: www.selectscience.net www.phonedog.com www.mediabistro.com www.practicalecommerce.com/

How to **represent** a graph?

Conceptually.
Visually.
Programmatically.

How to **Represent** a Graph?

Visually



Adjacency matrix

Target node

Source node

	1	2	3	4
1	0	1	3	0
2	0	0	0	2
3	0	1	0	0
4	0	0	0	0

Adjacency list

1: 2, 3
2: 4
3: 2

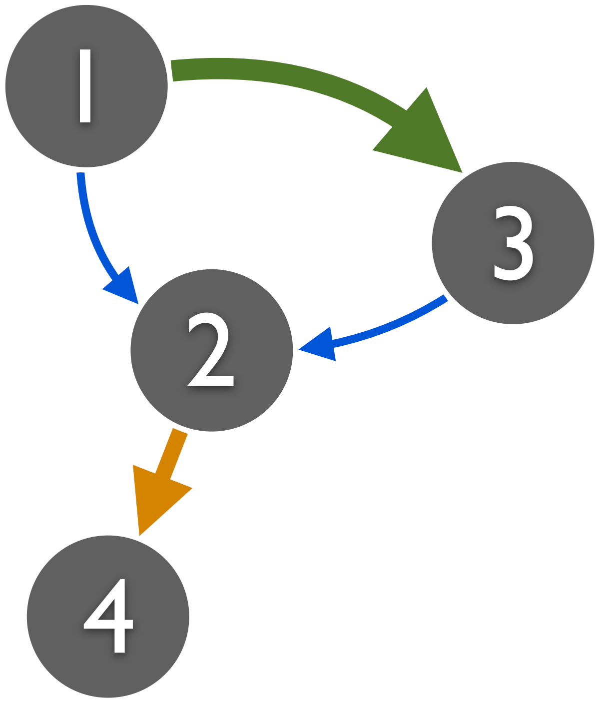
Edge list

1, 2, 1
1, 3, 3
2, 4, 2
3, 2, 1

- most common distribution format
- sometimes **painful** to parse when edges/nodes have many columns (some are text with double/single quotes, some are integers, some decimals, ...)

How to Represent a Graph?

Visually



Adjacency matrix

Target node

Source node

	1	2	3	4
1	0	1	3	0
2	0	0	0	2
3	0	1	0	0
4	0	0	0	0

Adjacency list

1: 2, 3
2: 4
3: 2

Edge list

1, 2, 1
1, 3, 3
2, 4, 2
3, 2, 1

Each node is often identified by a numeric ID. Why?

Assigning an ID to a node

- Use a “map” (Java) / “dictionary” (Python) / SQLite
- Same concept: given an entity/node (e.g., “Tom”) not seen before, assign a number to it
- Example of using SQLite to map names to IDs

Hidden column; SQLite automatically created for you

rowid	**name**
1	Tom
2	Sandy
3	Richard
4	Polo

How to use the node IDs?

Create an index for “name”. Then write a “join” query.

rowid	**name**
1	Tom
2	Sandy
3	Richard
4	Polo

source	**target**
Tom	Sandy
Polo	Richard

source	**target**
1	2
4	3

How to store “large” graphs?

How large is “large”?

What do you think?

- In what units? Thousands? Millions?

How do you measure a graph’s size?

- By ...

(Hint: highly subjective. And domain specific.)

Storing large graphs...

On your laptop computer

- SQLite
- Neo4j (GPL license)
  http://neo4j.com/licensing/

On a server

- MySQL, PostgreSQL, etc.
- Neo4j (?)

Storing large graphs...

With a cluster

- **Titan** (on top of **HBase**), S2Graph — if you need real time read and write
- **Hadoop** (generic framework) — if batch processing is fine
- **Hama, Giraph**, inspired by Google's Pregel
- **FlockDB**, by Twitter
- Turri (Apple) / Dato / GraphLab

Storing large graphs on your computer

I like to use **SQLite**. Why? **Good enough for my use.**

- Easily handle up to **gigabytes**
  - Roughly **tens of millions** of nodes/edges (perhaps up to billions?). Very good! For **today's** standard.
- Very easy to maintain: **one** cross-platform file
- Has programming wrappers in numerous languages
  - C++, Java (Andriod), Python, Objective C (iOS),...
- Queries are so easy!
  e.g., find all nodes' degrees = 1 SQL statement
- Bonus: SQLite even supports **full-text search**
- Offline application support (iPad)

SQLite graph database schema

Simplest schema:

```
edges(source_id, target_id)
```

More sophisticated (flexible; lets you store more things):

```
CREATE TABLE nodes (
   id INTEGER PRIMARY KEY,
   type INTEGER DEFAULT 0,
   name VARCHAR DEFAULT '');

CREATE TABLE edges (
   source_id INTEGER,
   target_id INTEGER,
   type INTEGER DEFAULT 0,
   weight FLOAT DEFAULT 1,
   timestamp INTEGER DEFAULT 0,
   PRIMARY KEY(source_id, target_id, timestamp));
```

[Side note; you already did this in HW1]

Full-Text Search (FTS) on SQLite

http://www.sqlite.org/fts3.html

Very simple. Built-in. Only needs 3 lines of commands.

- **Create** FTS table (index)

```
CREATE VIRTUAL TABLE critics_consensus USING
fts4(consensus);
```

- **Insert** text into FTS table

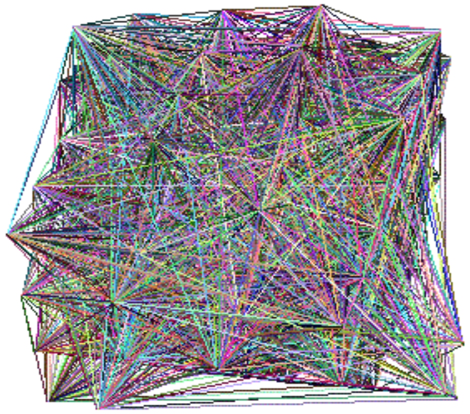
```
INSERT INTO critics_consensus SELECT
critics_consensus FROM movies;
```

- **Query** using the “match” keyword

```
SELECT * FROM critics_consensus WHERE consensus
MATCH 'funny OR horror';
```

SQLite originally developed by Google engineers

I have a graph dataset. Now what?



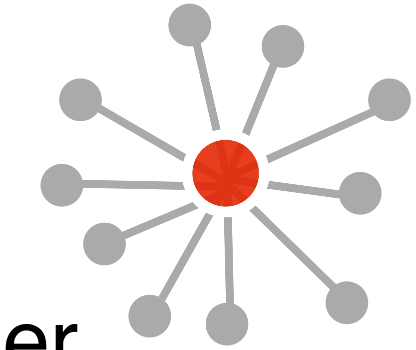
Yuck!

Analyze it! Do “**data mining**” or “**graph mining**”.

How does it “look like”? Visualize it if it’s small.

Does it follow any expected patterns?
Or does it *not* follow some expected patterns (outliers)?

- Why does this matter?
- If we know the **patterns** (models), we can do **prediction**, **recommendation**, etc.
  e.g., is Alice going to “friend” Bob on Facebook?
  People often buy beer and diapers together.
- **Outliers** often give us **new insights**
  e.g., telemarketer’s “friends” don’t know each other



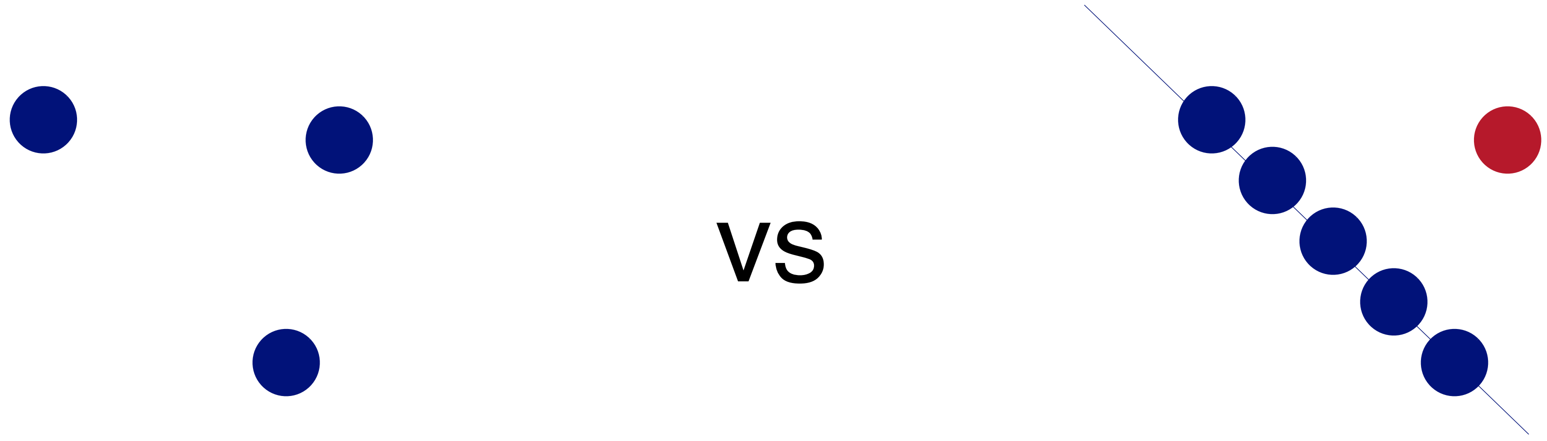
Finding patterns & outliers in graphs

Outlier/Anomaly detection

- To spot them, **we need to find patterns first**
- Anomalies = things that do not fit the patterns

To effectively do this, we need large datasets

- patterns and anomalies don’t show up well in small datasets



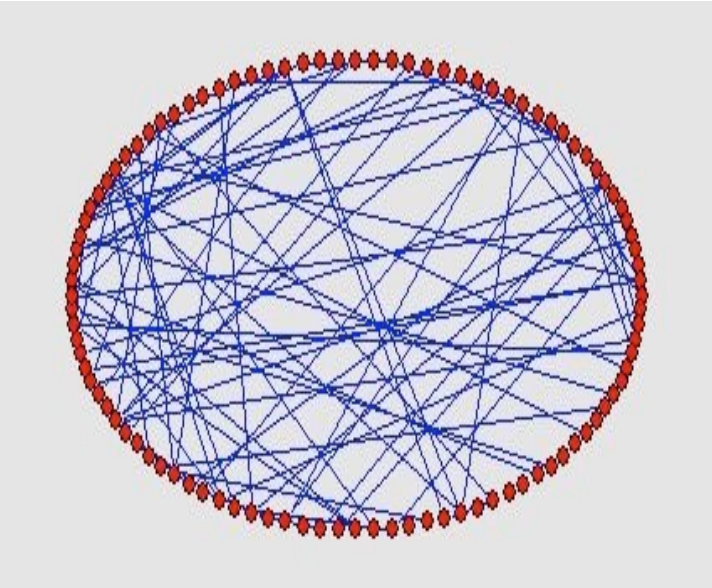
Are real graphs random?

Random graph (Erdos-Renyi)
100 nodes, avg degree = 2

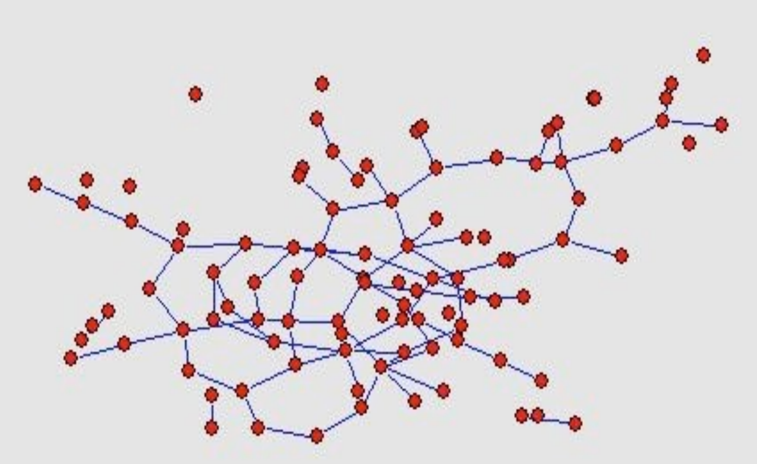
http://en.wikipedia.org/wiki/Erdős–Rényi_model

No obvious patterns

Before layout



After layout



Graph and layout generated with pajek

http://vlado.fmf.uni-lj.si/pub/networks/pajek/

Laws and patterns

Laws and patterns

- Are real graphs random?

Laws and patterns

- Are real graphs random?

Laws and patterns

- Are real graphs random?

- A: NO!!!
  - Diameter (longest shortest path)
  - in- and out- degree distributions
  - other (surprising) patterns
  - **So, let’s look at the data**

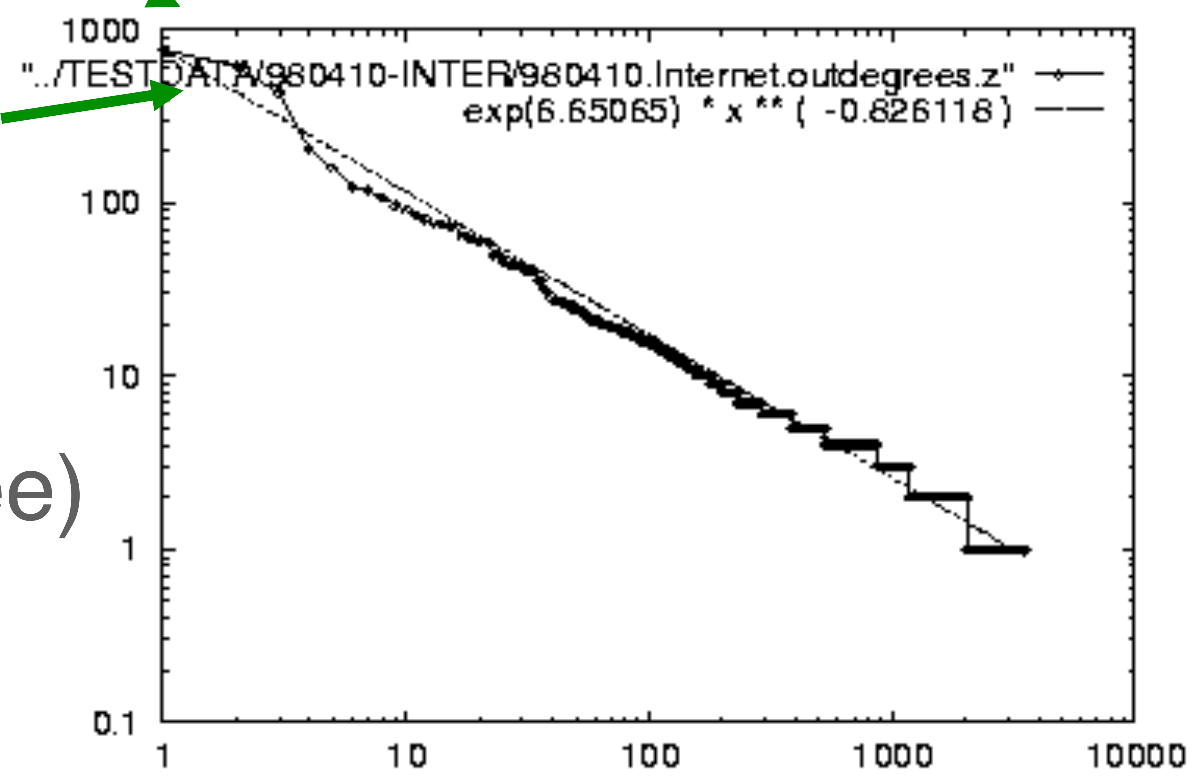
Power Law in Degree Distribution

Faloutsos, Faloutsos, Faloutsos [SIGCOMM99]
Seminal paper. Must read!

internet domains

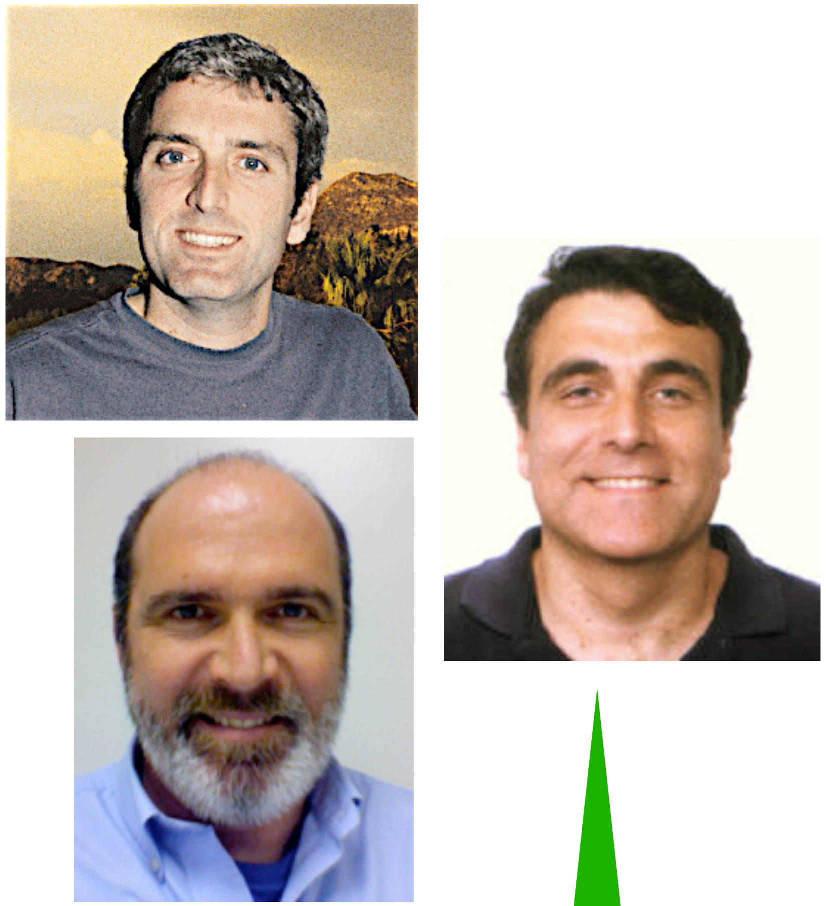
att.com

ibm.com



log(degree)

log(rank)



Christos was Polo's advisor

Zipf's law: the frequency of any item is **inversely proportional** to the item's rank (when ranked by decreasing frequency)

http://en.wikipedia.org/wiki/Zipf%27s_law

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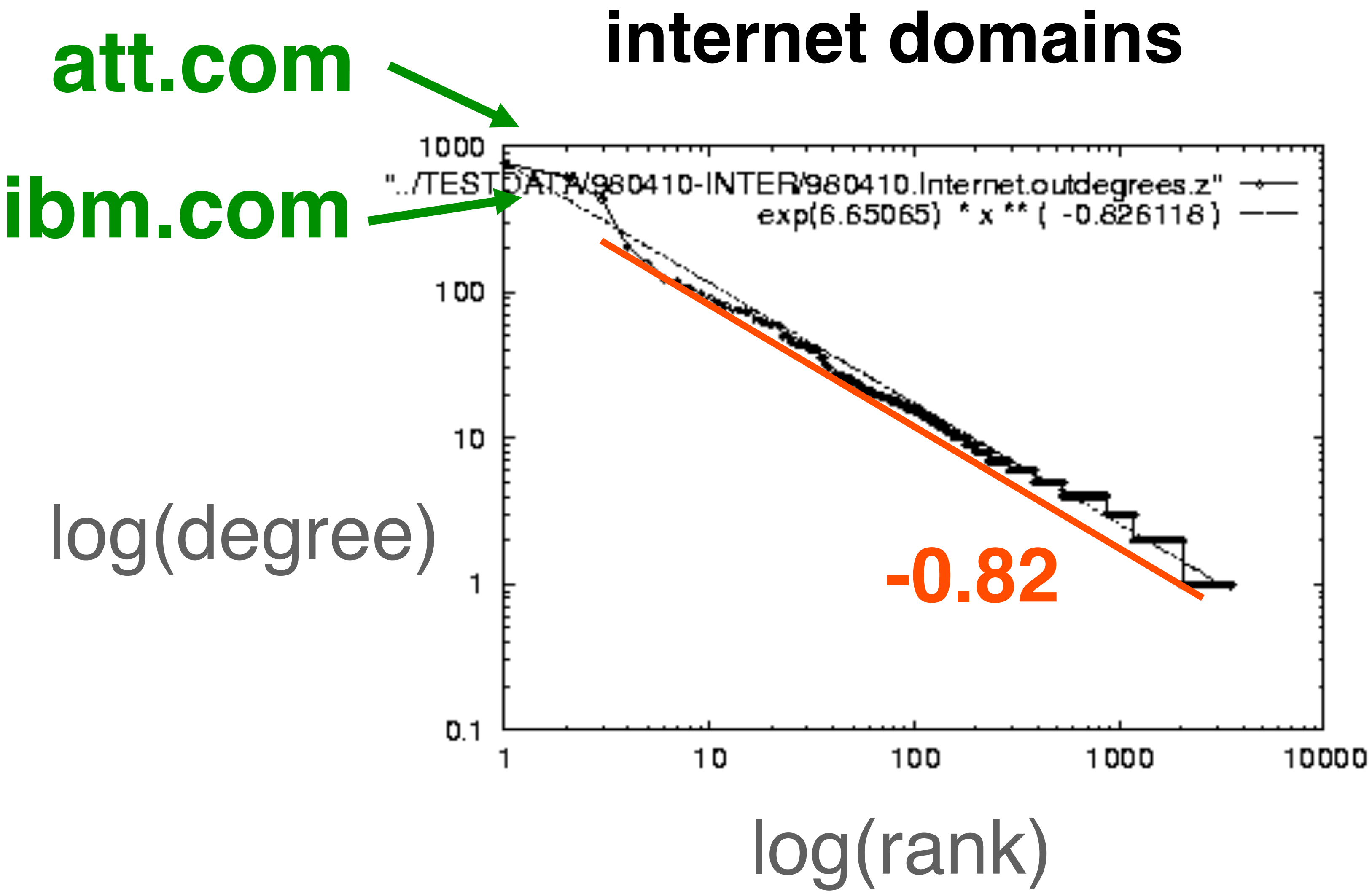

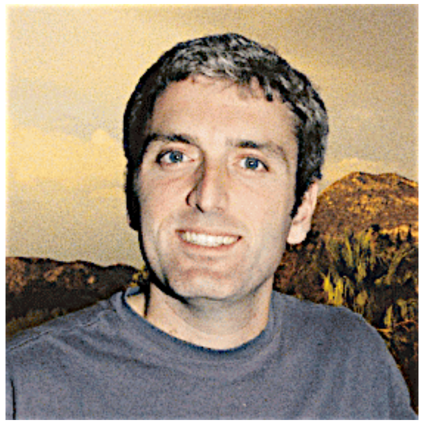

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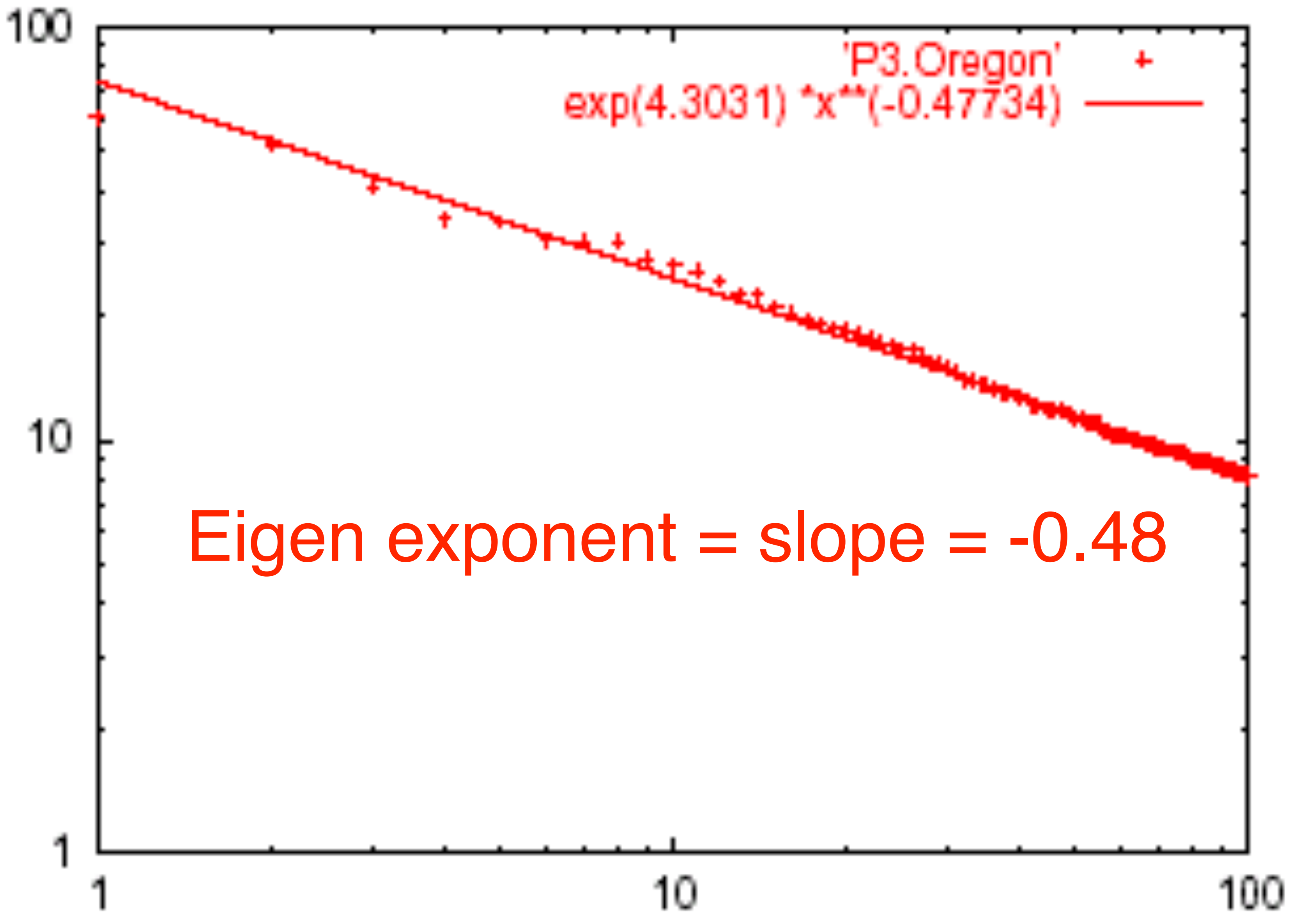
http://en.wikipedia.org/wiki/Zipf%27s_law

Power Law in Eigenvalues of Adjacency Matrix



Eigenvalue

Eigen exponent = slope = -0.48

Rank of decreasing eigenvalue

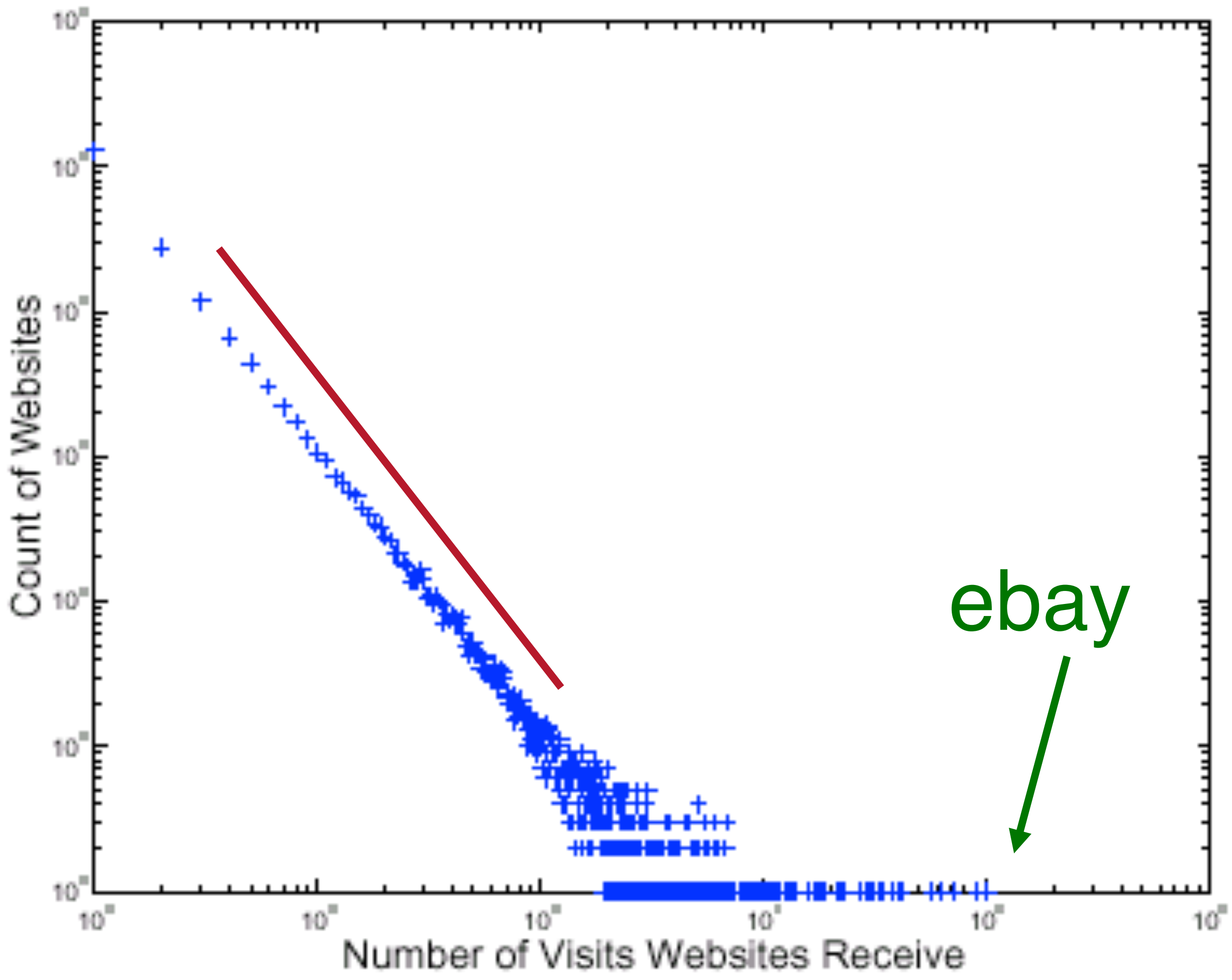
How about graphs from other domains?

More Power Laws

- Web hit counts
  [Alan L. Montgomery and Christos Faloutsos]

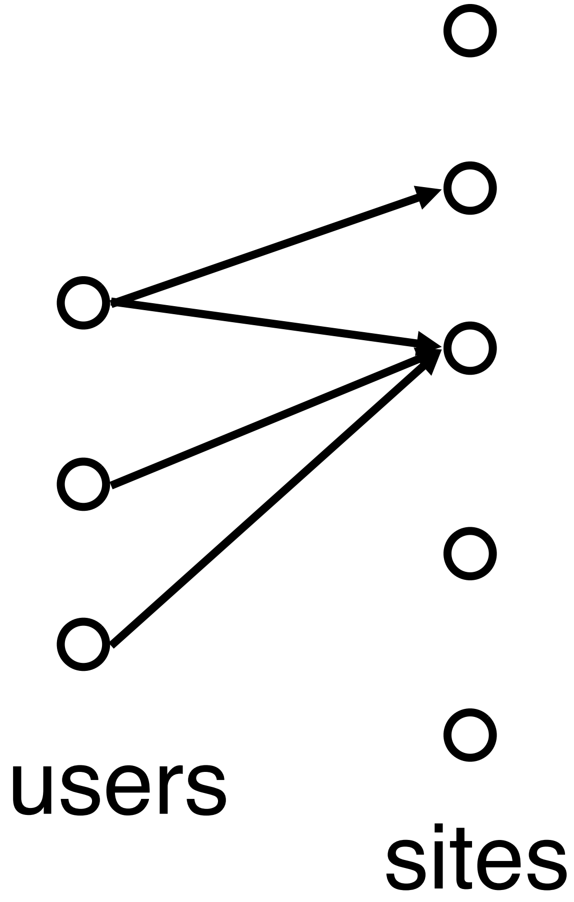
Web Site Traffic

log(#website)



ebay

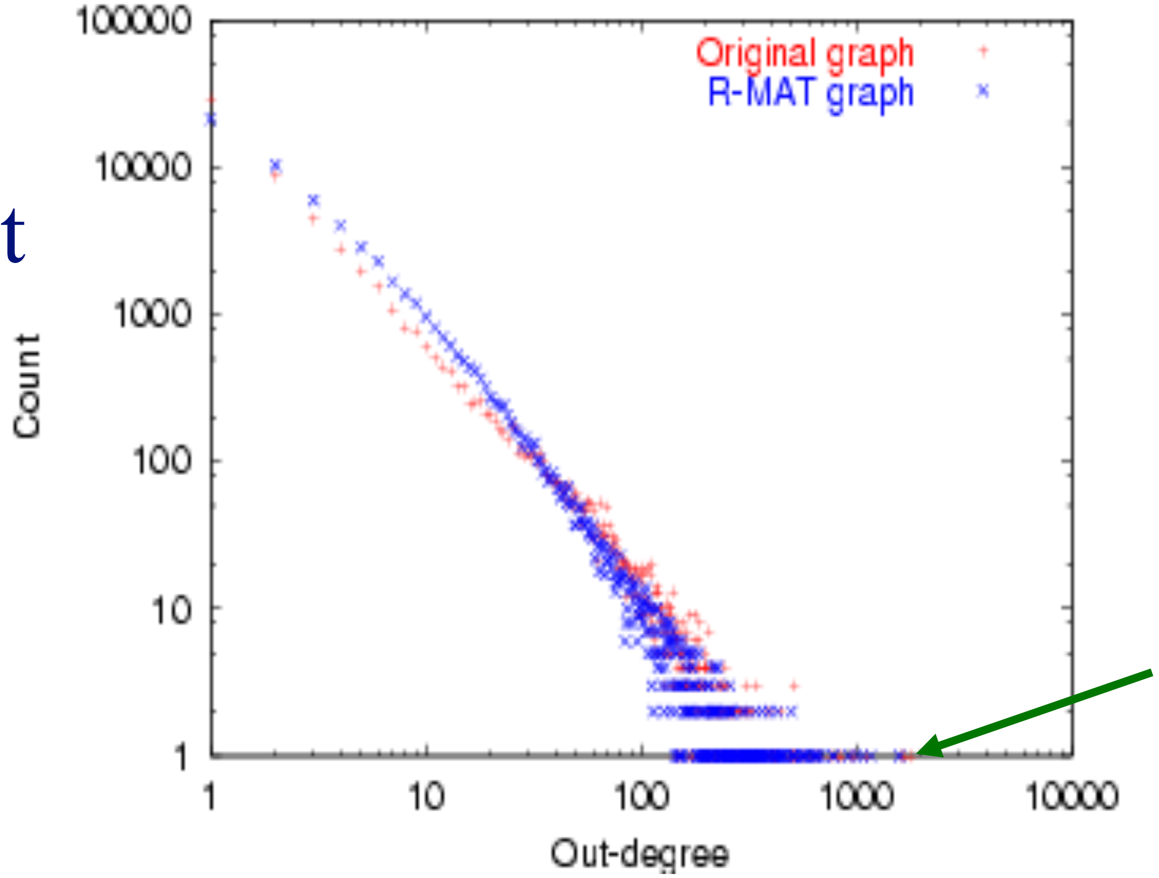
log(#website visit)



epinions.com

- who-trusts-whom [Richardson + Domingos, KDD 2001]

count



trusts-2000-people user

(out) degree

And numerous more

- # of sexual contacts
- Income [Pareto] – 80-20 distribution
- Duration of downloads [Bestavros+]
- Duration of UNIX jobs
- File sizes
- …

Any other ‘laws’?

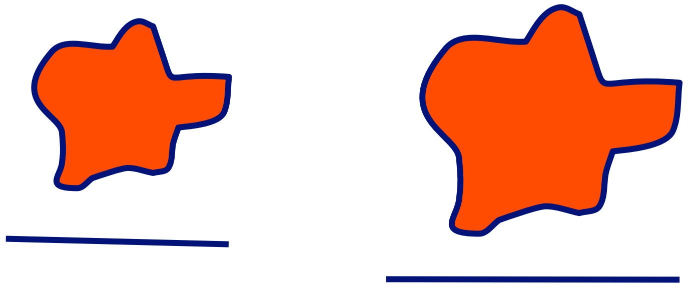
- Yes!
- Small diameter (~ constant!) –
  - six degrees of separation / ‘Kevin Bacon’
  - small worlds [Watts and Strogatz]

Problem: Time evolution

- Jure Leskovec (CMU -> Stanford)
- Jon Kleinberg (Cornell)
- Christos Faloutsos (CMU)

Evolution of the Diameter

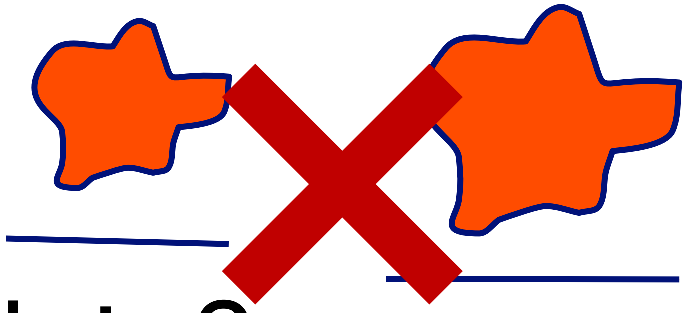
- Prior work on Power Law graphs hints at slowly growing diameter:
  - diameter ~ O(log N)
  - diameter ~ O(log log N)



- What is happening in real data?

Evolution of the Diameter

- Prior work on Power Law graphs hints at slowly growing diameter:
  - diameter ~ O(log N)
  - diameter ~ O(log log N)

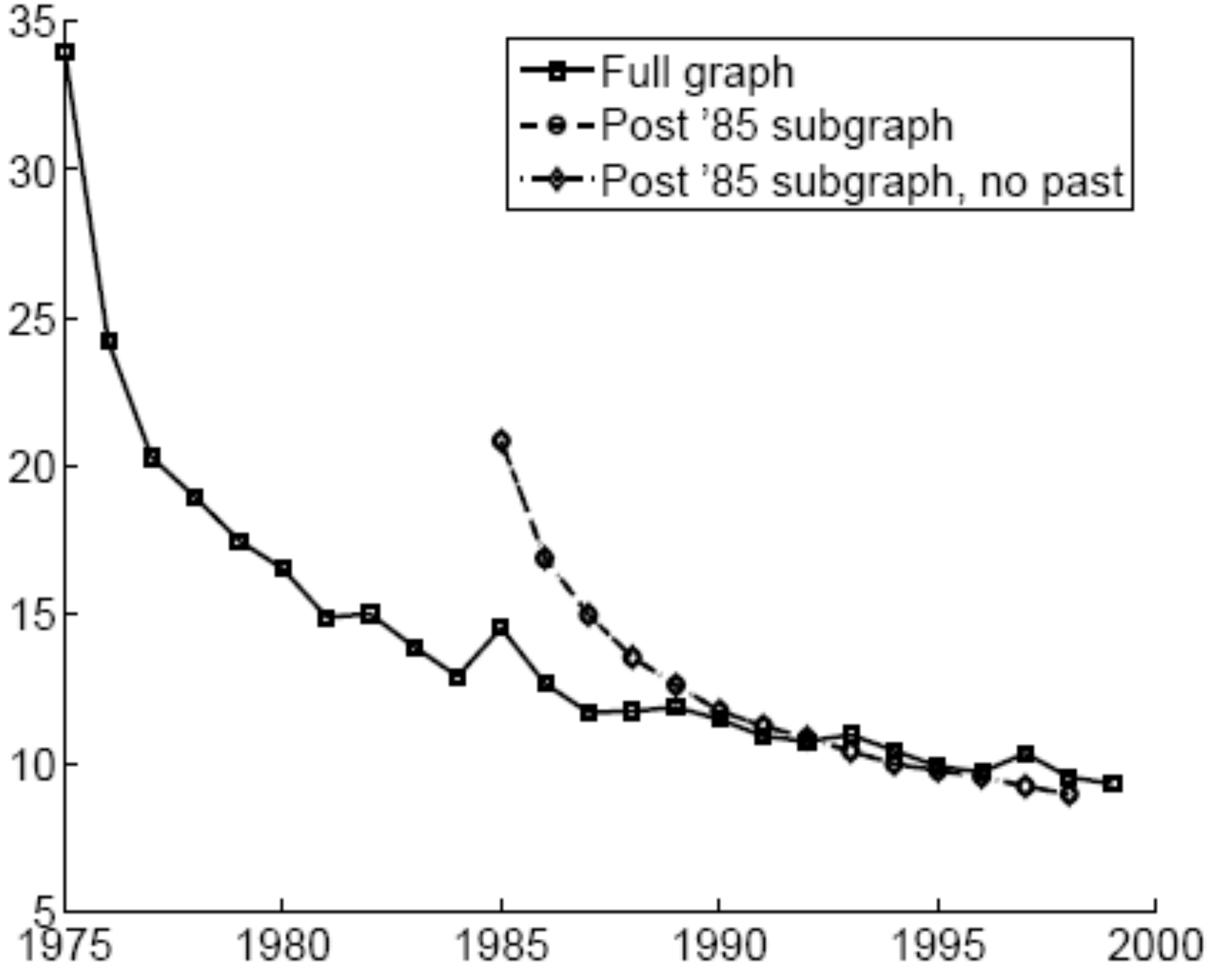


- What is happening in real data?
- Diameter shrinks over time

Diameter – Patents Network

- Patent citation network
- 25 years of data
- @1999
  - 2.9 M nodes
  - 16.5 M edges

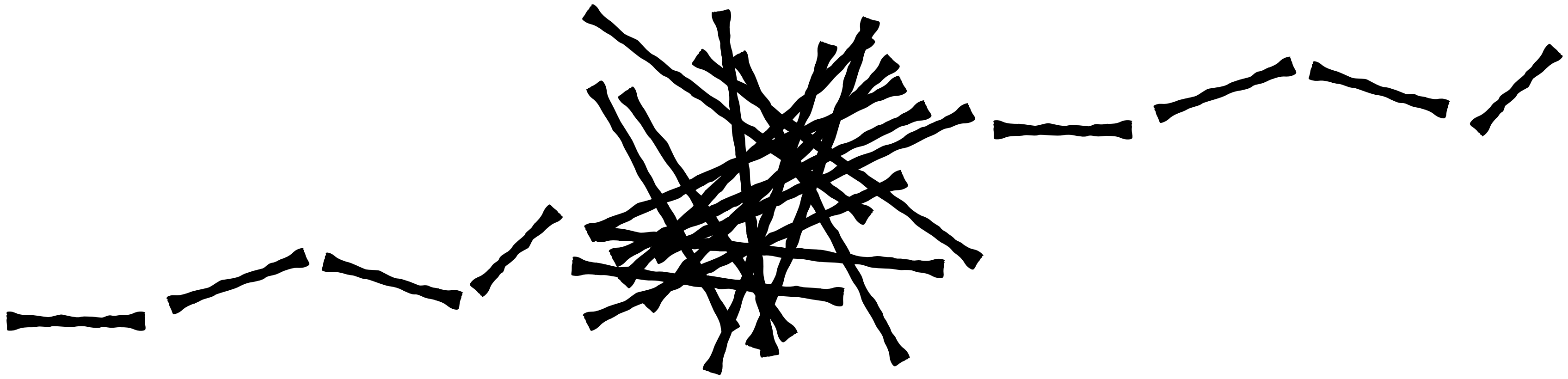
Effective diameter



Time (year)

Why **Effective Diameter**?

The **maximum** diameter is susceptible to **outliers**



So, we use **effective** diameter instead

- defined as the **minimum** number of hops in which **90% of connected node pairs** can reach each other

Evolution of #Node and #Edge

N(t) … nodes at time t

E(t) … edges at time t

Suppose that

$$N(t+1) = 2 * N(t)$$

Q: what is your guess for

$$E(t+1) =? 2 * E(t)$$

Evolution of #Node and #Edge

N(t) … nodes at time t

E(t) … edges at time t

Suppose that

$$N(t+1) = 2 * N(t)$$

Q: what is your guess for

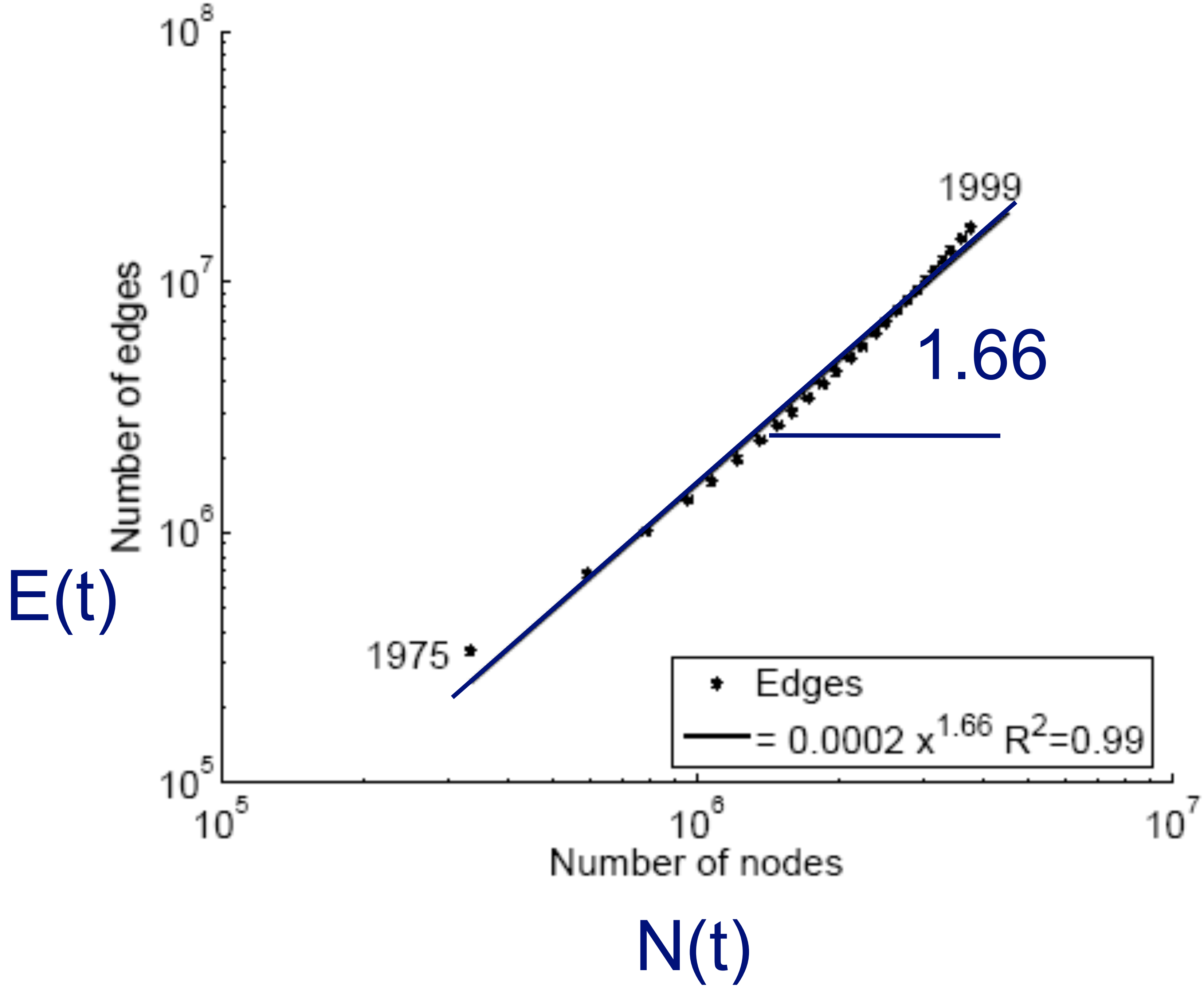
$$E(t+1) =? 2 * E(t)$$ 🚫

A: over-doubled!

But obeying the “Densification Power Law” 😯

Densification – Patent Citations

- Citations among patents granted
- @1999
  - 2.9 M nodes
  - 16.5 M edges
- Each year is a datapoint

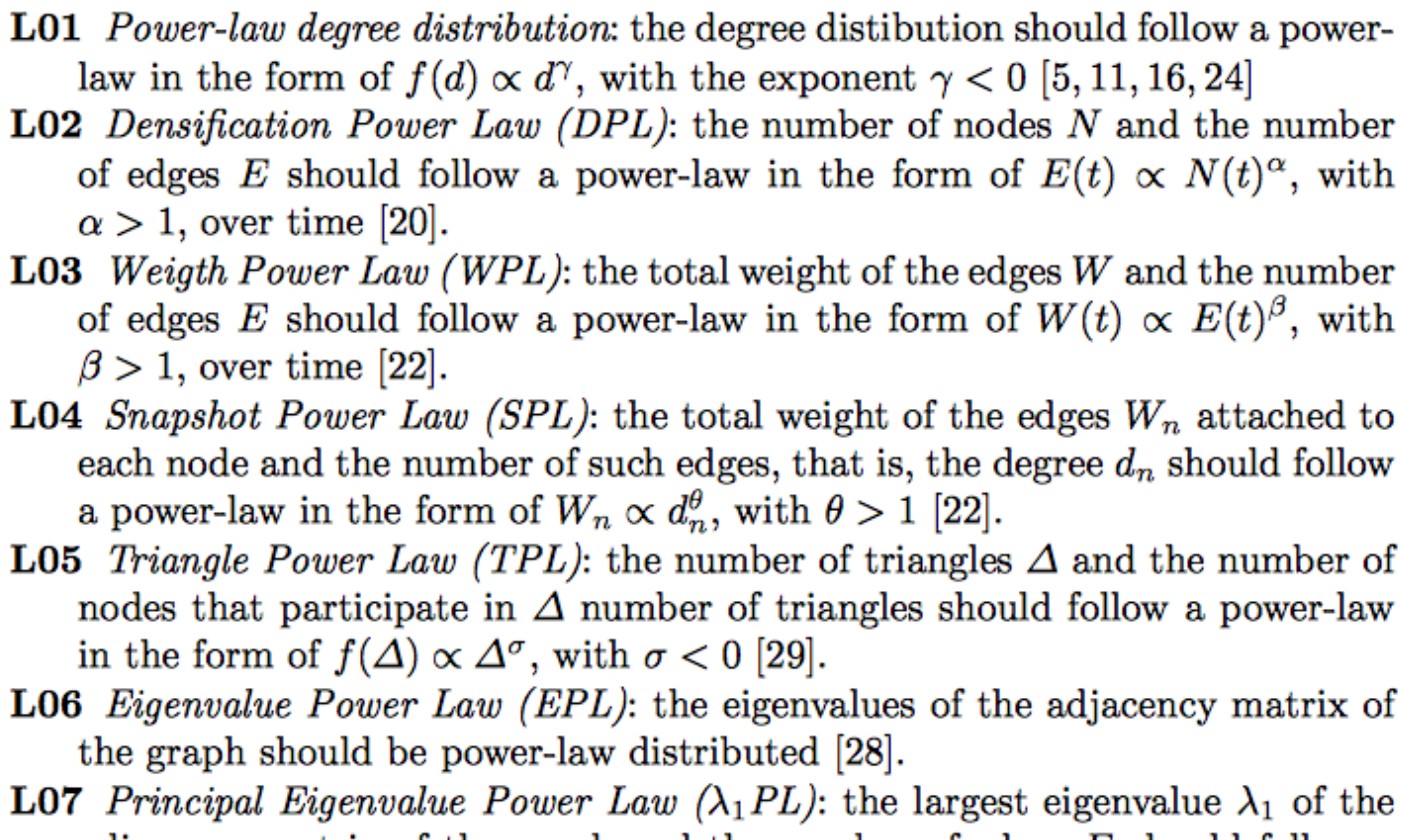


So many laws!

There will be more to come...

To date, there are **11 (or more) laws**

- RTG: A Recursive Realistic Graph Generator using Random Typing [Akoglu, Faloutsos]



L01 *Power-law degree distribution*: the degree distibution should follow a power-law in the form of $f(d) \propto d^{\gamma}$, with the exponent $\gamma < 0$ [5, 11, 16, 24]

L02 *Densification Power Law (DPL)*: the number of nodes N and the number of edges E should follow a power-law in the form of $E(t) \propto N(t)^{\alpha}$, with $\alpha > 1$, over time [20].

L03 *Weigth Power Law (WPL)*: the total weight of the edges W and the number of edges E should follow a power-law in the form of $W(t) \propto E(t)^{\beta}$, with $\beta > 1$, over time [22].

L04 *Snapshot Power Law (SPL)*: the total weight of the edges W_n attached to each node and the number of such edges, that is, the degree d_n should follow a power-law in the form of $W_n \propto d_n^{\theta}$, with $\theta > 1$ [22].

L05 *Triangle Power Law (TPL)*: the number of triangles Δ and the number of nodes that participate in Δ number of triangles should follow a power-law in the form of $f(\Delta) \propto \Delta^{\sigma}$, with $\sigma < 0$ [29].

L06 *Eigenvalue Power Law (EPL)*: the eigenvalues of the adjacency matrix of the graph should be power-law distributed [28].

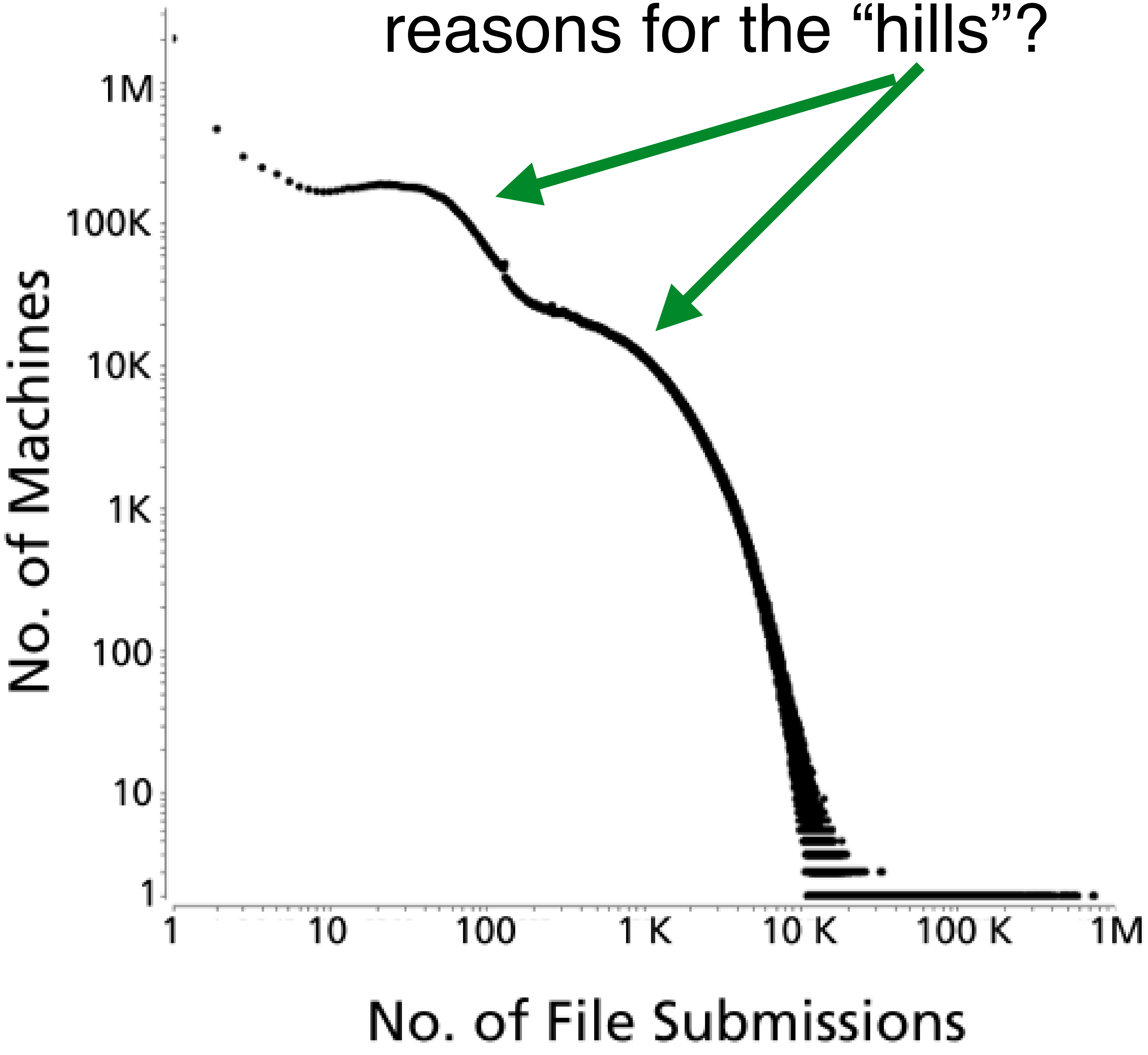
L07 *Principal Eigenvalue Power Law ($\lambda_1 PL$)*: the largest eigenvalue λ_1 of the

So many laws!

What should you do?

- **Try as many distributions as possible** and see if your graph fits them.
- **If it doesn't, find out the reasons.** Sometimes it's due to errors/problems in the data; sometimes, it signifies some new patterns!

What might be the reasons for the “hills”?



Polonium: Tera-Scale Graph Mining and Inference for Malware Detection [Chau, et al]