CX4242:

Classification Key Concepts

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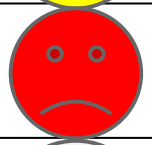
How will I rate "Chopin's 5th Symphony"?

Songs	**Like?**
Some nights	
Skyfall	
Comfortably numb	
We are young	
...	...
...	...
Chopin's 5th	???

Classification

What tools do you need for classification?

1. **Data** $S = \{(x_i, y_i)\}_{i = 1,\ldots,n}$
   - x_i : data example with d attributes
   - y_i : label of example (what **you** care about)

2. Classification **model** $f_{(a,b,c,\ldots.)}$ with some parameters *a, b, c,…*
3. **Loss function** *L(y, f(x))*
   - how to penalize mistakes

Terminology Explanation

data example = data instance
attribute = feature = dimension
label = target attribute

- **Data** $S = \{(x_i, y_i)\}_{i=1,\dots,n}$
  - x_i : data example with d attributes
  - y_i : label of example

$$x_i = (x_{i1}, \dots, x_{id})$$

Song name	**Artist**	**Length**	**...**	**Like?**
Some nights	Fun	4:23	...	
Skyfall	Adele	4:00	...	
Comf. numb	Pink Fl.	6:13	...	
We are young	Fun	3:50	...	
...	...	...	...	...
...	...	...	...	...
Chopin's 5th	Chopin	5:32	...	??

What is a “model”?

“a simplified representation of reality created to serve a purpose” *Data Science for Business*

Example: maps are abstract models of the physical world

There can be many models!!

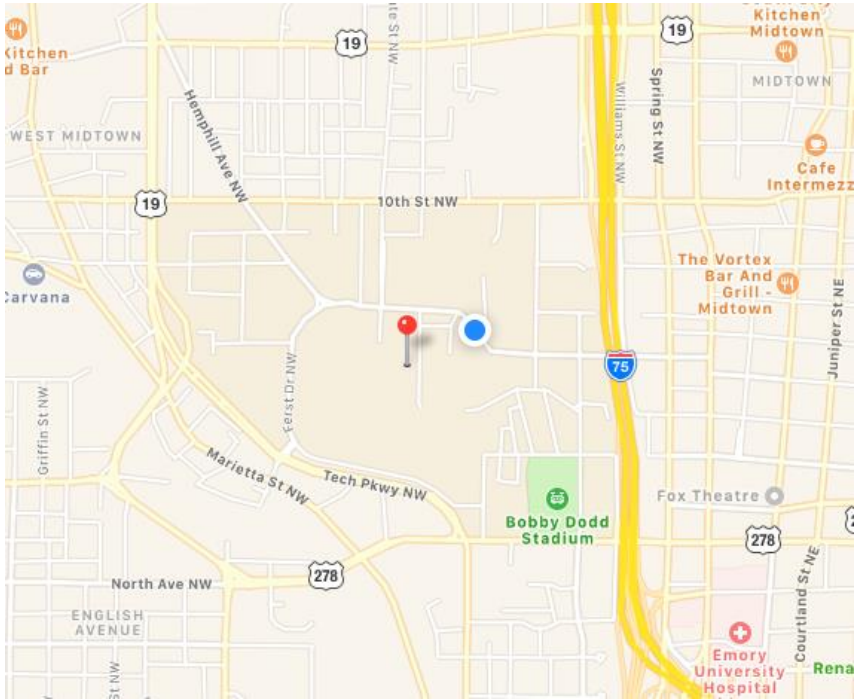
(Everyone sees the world differently, so each of us has a different model.)

In data science, a model is **formula to estimate what you care about**. The formula may be mathematical, a set of rules, a combination, etc.

Training a classifier = building the "model"

How do you learn appropriate values for parameters *a, b, c, ...* ?

Analogy: how do you know your map is a "good" map of the physical world?



Classification loss function

Most common loss: **0-1 loss function**

$$L_{0-1}(y, f(x)) = \mathbb{I}(y \neq f(x))$$

More general loss functions are defined by a *m x m* cost matrix *C* such that

$$L(y, f(x)) = C_{ab}$$

where $y = a$ and $f(x) = b$

Class	**P0**	**P1**
T0	0	C_{10}
T1	C_{01}	0

T0 (true class 0), **T1** (true class 1)

P0 (predicted class 0), **P1** (predicted class 1)

An ideal model should correctly estimate:

- known or seen data examples' labels
- unknown or unseen data examples' labels

Song name	Artist	Length	...	Like?
Some nights	Fun	4:23	...	
Skyfall	Adele	4:00	...	
Comf. numb	Pink Fl.	6:13	...	
We are young	Fun	3:50	...	
...	...	...	...	...
...	...	...	...	...
Chopin's 5th	Chopin	5:32	...	??

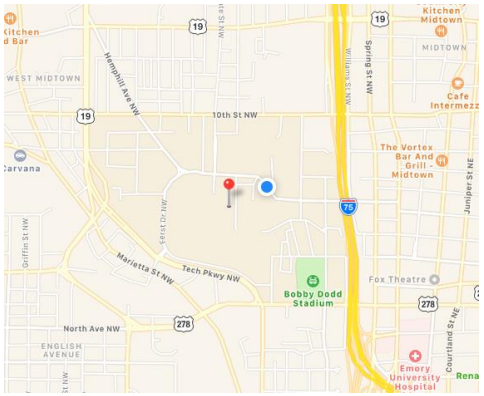
Training a classifier = building the "model"

Q: How do you learn appropriate values for parameters *a*, *b*, *c*, ... ?

(Analogy: how do you know your map is a "good" map?)

- $y_i = f_{(a,b,c,....)}(x_i)$, $i = 1, \ldots, n$
  - Low/no error on training data ("seen" or "known")
- $y = f_{(a,b,c,....)}(x)$, for any new x
  - Low/no error on test data ("unseen" or "unknown")

It is very easy to achieve perfect classification on training/seen/known data. Why?



If your model works really well for *training* data, but poorly for *test* data, your model is “overfitting”.

How to avoid overfitting?

Example: one run of *5-fold* cross validation

You should do a **few runs** and **compute the average**
(e.g., error rates if that's your evaluation metrics)

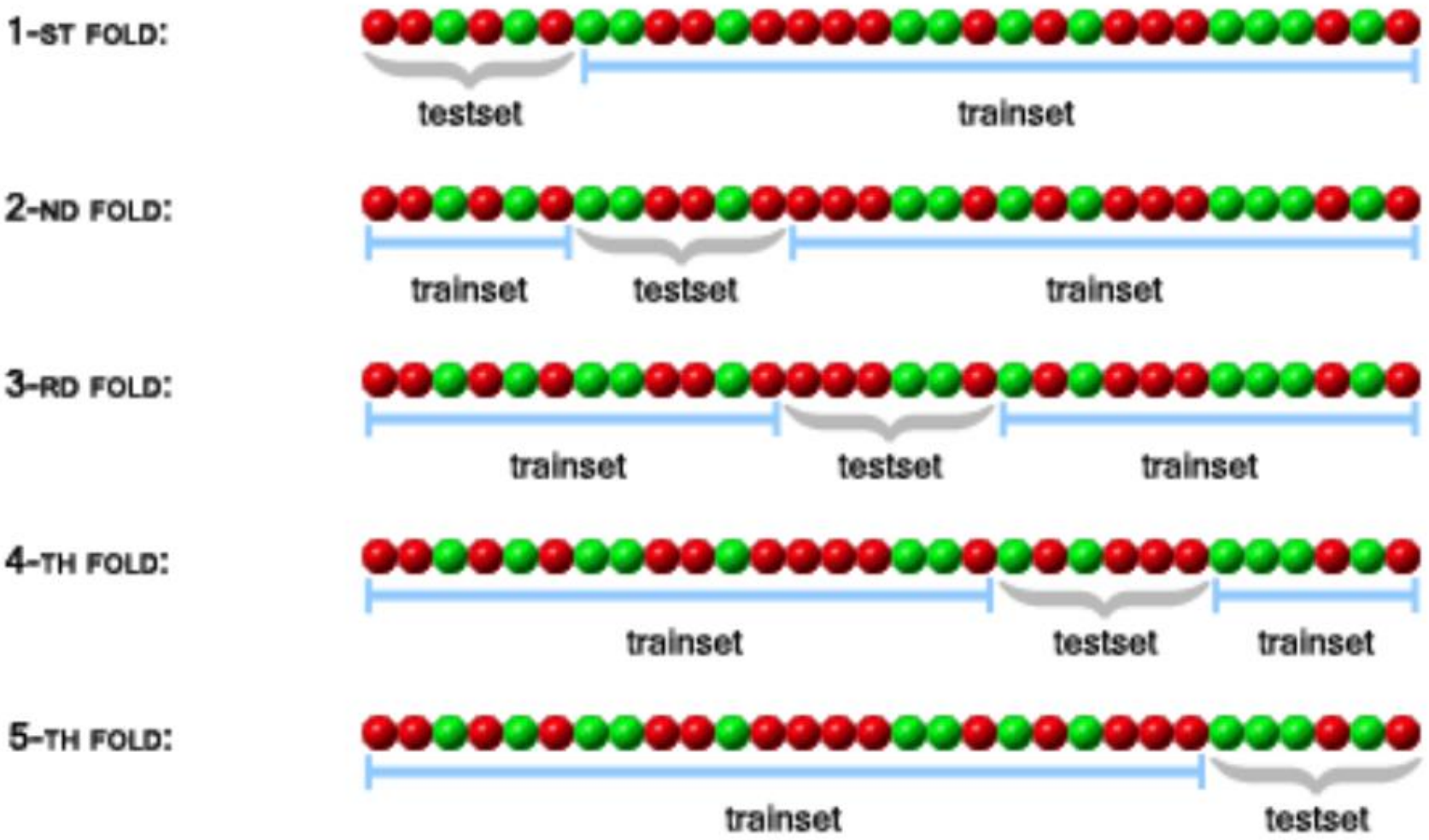
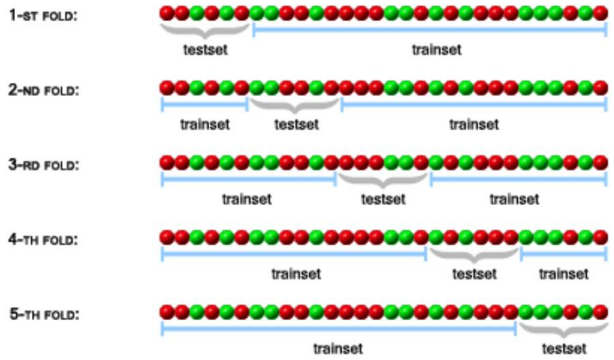


Image credit: http://stats.stackexchange.com/questions/1826/cross-validation-in-plain-english

Cross validation



1. Divide your data into n parts
2. Hold 1 part as “test set” or “hold out set”
3. Train classifier on remaining n-1 parts “training set”
4. Compute test error on test set
5. Repeat above steps n times, once for each n-th part
6. Compute the average test error over all n folds
   (i.e., cross-validation test error)

Cross-validation variations

K-fold cross-validation

- Test sets of size *(n / K)*
- K = 10 is most common (i.e., 10-fold CV)

Leave-one-out cross-validation (LOO-CV)

- test sets of size 1

Example:
k-Nearest-Neighbor classifier

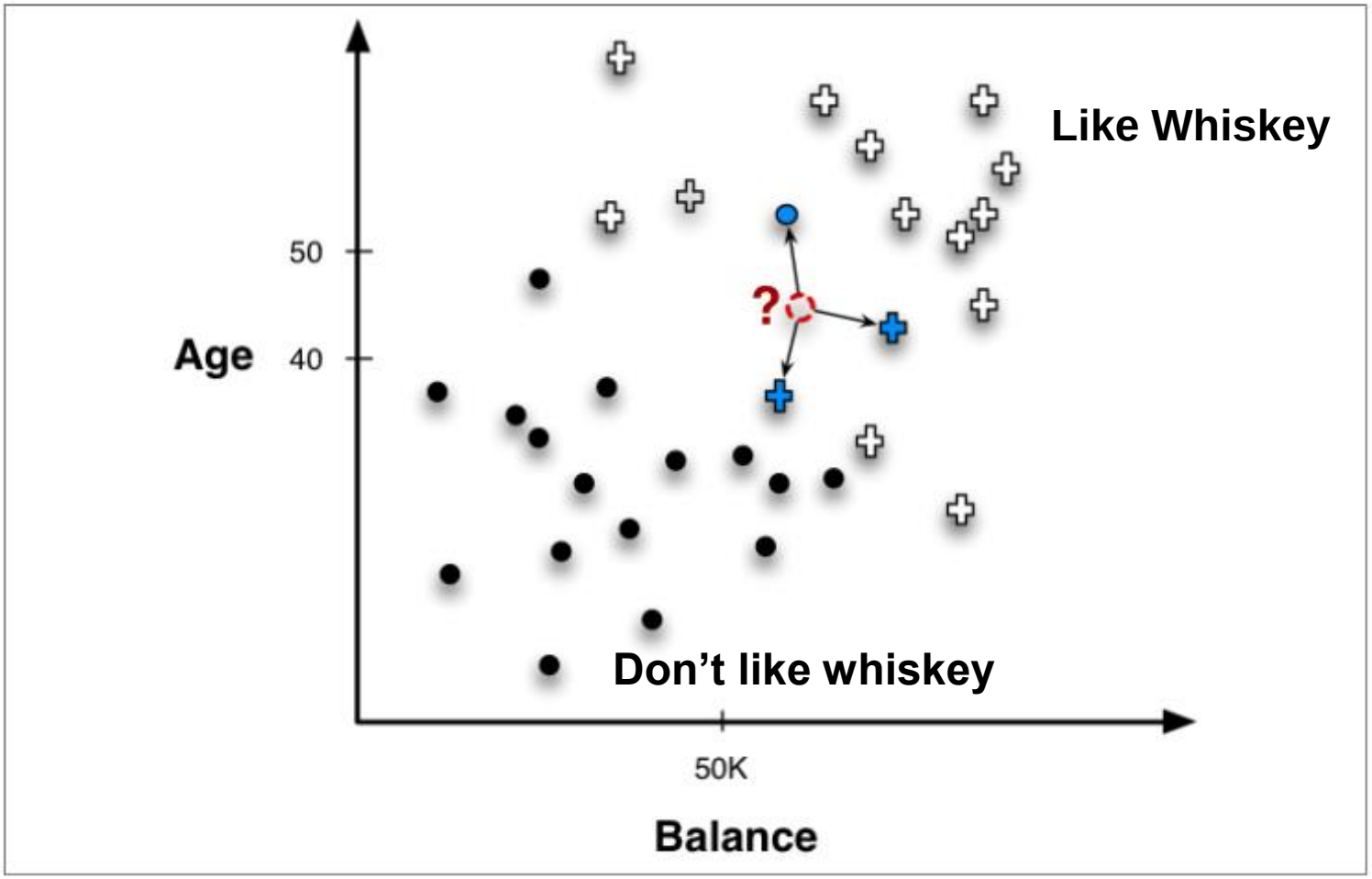


Figure 6-2. Nearest neighbor classification. The point to be classified, labeled with a question mark, would be classified + because the majority of its nearest (three) neighbors are +.

Image credit: Data Science for Business

But k-NN is so simple!

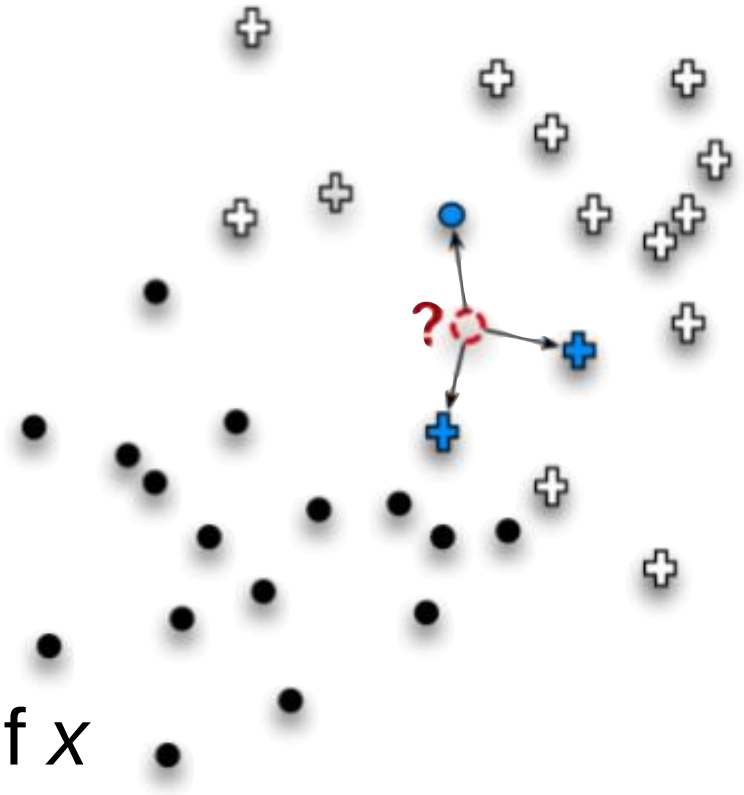
It can work really well! **Pandora** (acquired by SiriusXM) uses it or has used it: https://goo.gl/foLfMP
(from the book “Data Mining for Business Intelligence”)

Image credit: https://www.fool.com/investing/general/2015/03/16/will-the-music-industry-end-pandoras-business-mode.aspx

What are good models?

Simple (few parameters)	Effective	
Complex (more parameters)	Effective (if significantly more so than simple methods)	
Complex (many parameters)	Not-so-effective	

k-Nearest-Neighbor Classifier



The classifier:

f(x) = majority label of the *k* nearest neighbors (NN) of *x*

Model parameters:

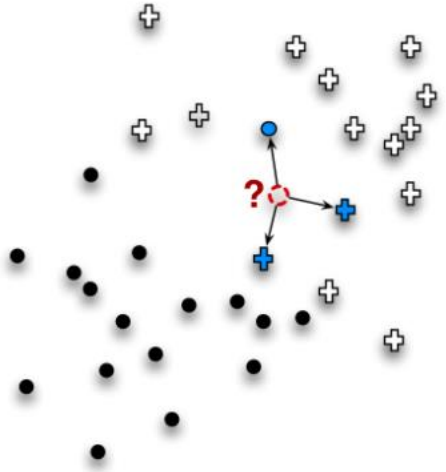
- Number of neighbors *k*
- Distance/similarity function *d(.,.)*

k-Nearest-Neighbor Classifier

If *k* and *d(.,.)* are fixed

Things to learn: ?

How to learn them: ?



If *d(.,.)* is fixed, but you can change *k*

Things to learn: ?

How to learn them: ?

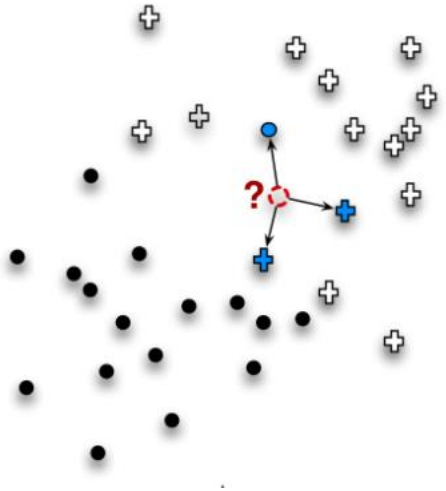
$$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$$

k-Nearest-Neighbor Classifier

If *k* and *d(.,.)* are fixed

Things to learn: Nothing

How to learn them: N/A



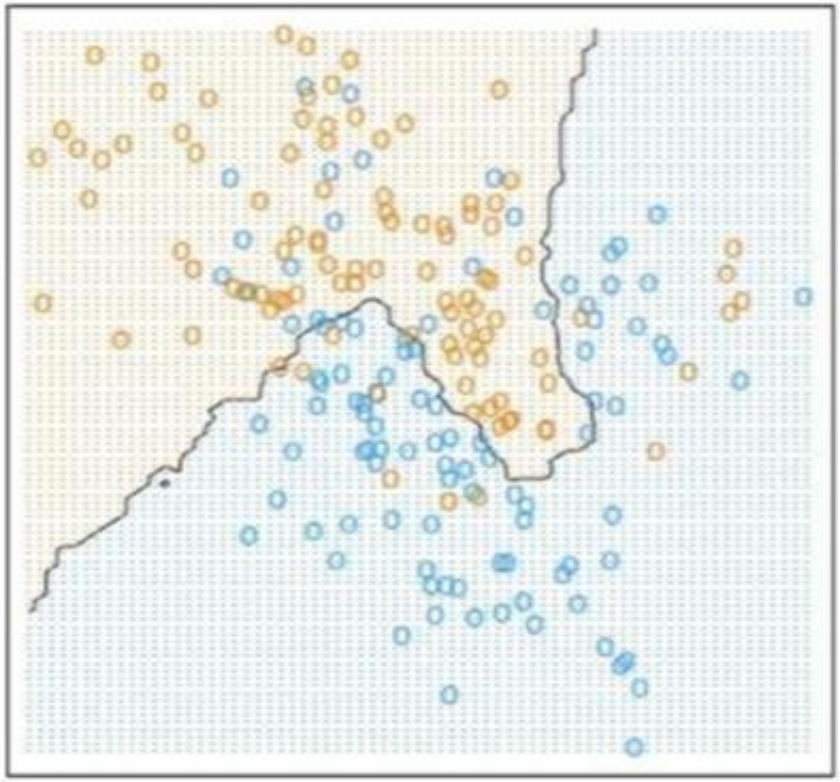
If *d(.,.)* is fixed, but you can change *k*

Selecting *k*: How?

How to find best k in k-NN?

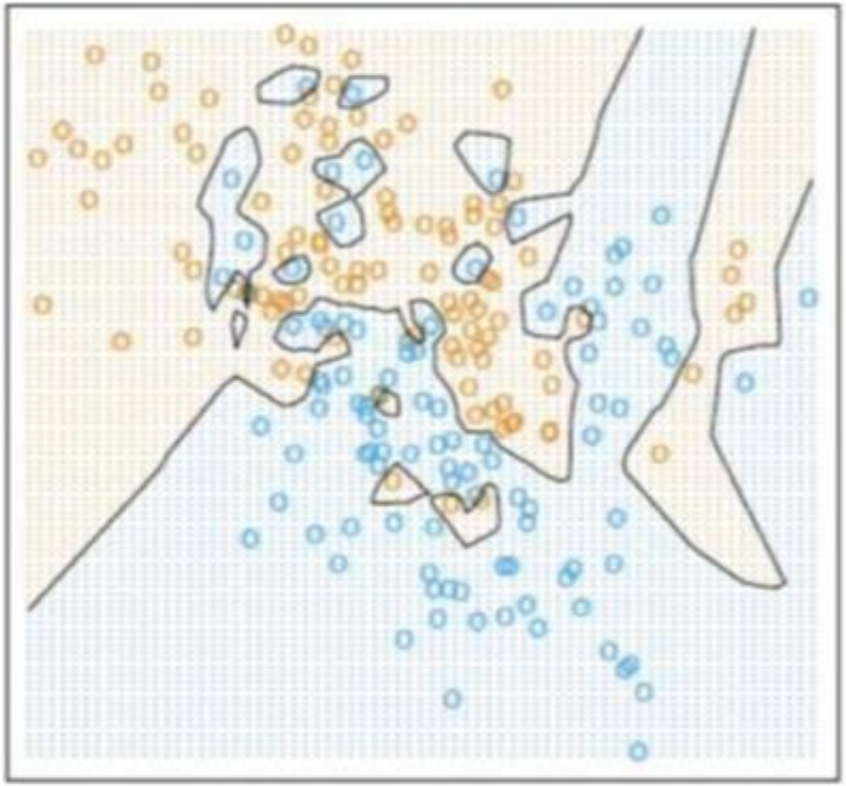
Use **cross validation (CV)**.

15-NN



Pretty good!

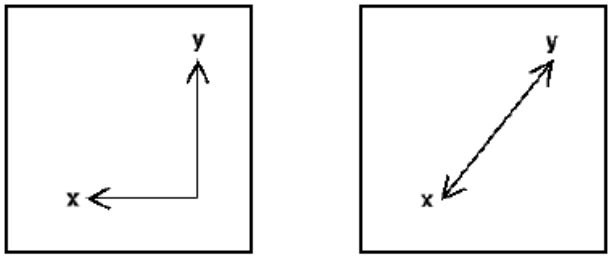
1-NN



Overfitted

k-Nearest-Neighbor Classifier

If *k* is fixed, but you can change *d(.,.)*



Possible distance functions:

- Euclidean distance: $\|x_i - x_j\|_2 = \sqrt{(x_i - x_j)^\top (x_i - x_j)}$
- Manhattan distance: $\|x_i - x_j\|_1 = \sum_{l=1}^{d} |x_{il} - x_{jl}|$
- …

$$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$$

Summary on k-NN classifier

- Advantages
  - Little learning (unless you are learning the distance functions)
  - Quite powerful in practice (and has theoretical guarantees)
- Caveats
  - Computationally expensive at test time

Reading material:

- The Elements of Statistical Learning (ESL) book, Chapter 13.3
  https://web.stanford.edu/~hastie/ElemStatLearn/